Unit Impulse Function

Lesson #2
2CT.2,4,
3CT.2
Appendix A

Homework

- Complex numbers
  - Convert 1+j1 to its magnitude/angle representation (phasor)
  - Convert 1/(1+j1) to a phasor
  - Draw $e^{j\omega t}$ and $e^{j(\omega t+\alpha)}$ in the complex plane
  - For the series R-L circuit in class, calculate the voltage across the inductor.
  - Appendix A.4, A.7
- Unit Impulse and Unit Step Functions
  - Using unit step functions, construct a single pulse of magnitude 10 starting at t=5 and ending at t=10.
  - Repeat problem 1) with 2 pulses where the second is of magnitude 5 starting at t=15 and ending at t=25.
  - Is the unit step function a bounded function?
  - Is the unit impulse function a bounded function?
  - 2CT.2.4a,b

Homework Answers

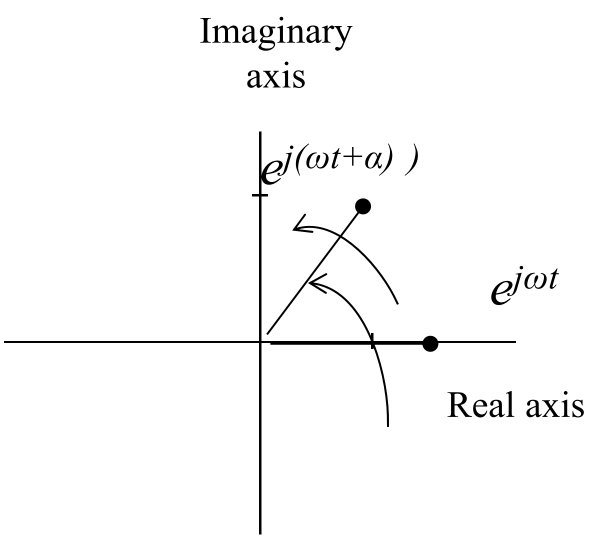
- Complex numbers
  - Convert *1+j1* to its magnitude/angle representation (phasor)
  - Convert *1/(1+j1)* to a phasor

$$a)\ 1+1j = \sqrt{1^2+1^2}\angle\tan^{-1}(1) = \sqrt{2}\angle 45^\circ$$

$$b)\ \frac{1}{1+1j} = \frac{1}{\sqrt{1^2+1^2}\angle\tan^{-1}(1)} = \frac{1}{\sqrt{2}\angle 45^\circ} = \frac{1}{\sqrt{2}}\angle -45^\circ$$

$$b)\ \frac{1}{1+1j} = \frac{1}{1+1j}\times\frac{1-1j}{1-1j} = \frac{1-1j}{1+1} = \frac{1}{2}1-1j = \frac{1}{2}\sqrt{2}\angle\tan^{-1}(-1) = \frac{1}{\sqrt{2}}\angle -45^\circ$$

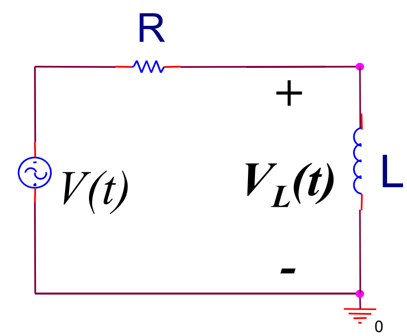
  - Draw $e^{j\omega t}$ and $e^{j(\omega t+\alpha)}$ in the complex plane



2 Rotating Unit Vectors at rate ω with one leading the other by α degrees

Homework Answers

- Complex numbers
  - For the series R-L circuit in class, calculate the voltage across the inductor.



$$V(t) \Rightarrow \mathbf{V} = A\angle 0;\ I(t) \Rightarrow \mathbf{I}; V_L(t) \Rightarrow \mathbf{V_L}$$

$$\mathbf{I} = \frac{A}{\sqrt{R^2 + (\omega L)^2}} \angle -\tan^{-1}(\omega L / R)$$

$$\mathbf{V_L} = \mathbf{I} Z_L = j\omega L \times \frac{A}{\sqrt{R^2 + (\omega L)^2}} \angle -\tan^{-1}(\omega L / R)$$

$$= \frac{A\omega L}{\sqrt{R^2 + (\omega L)^2}} \angle \{\frac{\pi}{2} - \tan^{-1}(\omega L / R)\}$$

$$V_L(t) = \frac{A\omega L}{\sqrt{R^2 + (\omega L)^2}} \cos[\omega t + \frac{\pi}{2} - \tan^{-1}(\omega L / R)]$$

A4, A7

$$A4)\ (1+j)^{0.5} = (\sqrt{2}e^{j\frac{\pi}{4}})^{0.5} = (\sqrt{2})^{0.5}e^{j\frac{\pi}{4}(0.5)} = 2^{0.25}e^{j\frac{\pi}{8}} = 1.19e^{j\frac{\pi}{8}} = 1.098 + j0.4551$$

$$(1+j)^{0.5} = (\sqrt{2}e^{j(\frac{\pi}{4}+2\pi)})^{0.5} = (\sqrt{2})^{0.5}e^{j(\frac{\pi}{4}+2\pi)(0.5)} = 2^{0.25}e^{j(\frac{\pi}{8}+\pi)} = 1.19e^{j\frac{9\pi}{8}} = -1.098 - j0.4551$$

$$A7)\ \Re\{H(F)e^{j2\pi Ft}\} = \Re\{|H(F)|e^{j\angle H(F)}e^{j2\pi Ft}\} = \Re\{|H(F)|e^{j2\pi Ft+\angle H(F)}\} =$$

$$\Re\{|H(F)|\cos(2\pi Ft + \angle H(F)) + j|H(F)|\sin(2\pi Ft + \angle H(F))\} = |H(F)|\cos(2\pi Ft + \angle H(F))$$

Homework Answers

- Unit Impulse and Unit Step Functions
  - Using unit step functions, construct a single pulse of magnitude 10 starting at t=5 and ending at t=10.
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  - Is the unit step function a bounded function?
  - Is the unit impulse function a bounded function?

$$f(t) = 10[u(t-5) - u(t-10)]$$

$$g(t) = f(t) + 5[u(t-15) - u(t-25)]$$

$$\int_{-\infty}^{\infty} |u(t)|\, dt = \int_{-\infty}^{0} u(t) dt + \int_{0}^{\infty} 1 dt = t\Big|_0^{\infty} = \infty - 0 = \infty \text{ Not Bounded}$$

$$\int_{-\infty}^{\infty} |\delta(t)|\, dt = \int_{0-}^{0+} \delta(t) dt = 1 \quad \text{Bounded}$$

2CT.2.4

$$a)\ \int_{-\infty}^{\infty} \delta(3t)dt = \frac{1}{3};$$

Let $3t = x$ then $3dt = dx \Rightarrow dt = \frac{dx}{3}$

$$\int_{-\infty}^{\infty} \delta(3t)dt = \int_{-\infty}^{\infty} \delta(x)\frac{dx}{3} = \frac{1}{3}\int_{-\infty}^{\infty} \delta(x)dx$$

Since $\int_{-\infty}^{\infty} \delta(x)dx = 1;$ then

$$\int_{-\infty}^{\infty} \delta(3t)dt = \frac{1}{3}\int_{-\infty}^{\infty} \delta(x)dx = \frac{1}{3}$$

$b)$ From a) we can conclude $\int_{-\infty}^{\infty} \delta(at)dt = \frac{1}{a}$ since $at = x$ and $dt = \frac{dx}{a}$